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Team Composition for Perimeter Defense with Patrollers and Defenders

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Abstract— **We study team composition in the context of a multi-player pursuit-evasion game between intruders and defenders. The game has been previously studied assuming full state information on both teams. We extend this problem by requiring defenders to detect intruders using a limited sensor footprint before pursuit can begin. We simplify the policy synthesis for a heterogeneous team by decomposing the perimeter defense task into patrol and defense subtasks each performed by a homogeneous team. We derive a nonlinear relationship between the robot capabilities, the team sizes, and the overall defensive team performance. This interaction is then used to consider how to select the robots for each subtask when various types of robots with heterogeneous capabilities are available. We present how to accommodate parameter uncertainties and the coupling between the two subtasks in the team composition.**

I. Introduction

We consider a class of differential games in which the evader/intruder attempts to reach a target region without being captured by the pursuere/defender. The one vs. one version of the game was introduced by Isaacs as the *target-guarding* problem [1], [2]. The multi-player version has also been studied as *reach avoid games* [3], [4], [5], where the challenge is in designing the control policy in the high-dimensional state space.

In [6], we formulated the multi-player target-guarding problem in the context of perimeter defense. The intruder team tries to score by sending as many intruders as possible to the target, while the defender team tries to minimize this score by intercepting them. Cooperative strategies for both intruders and defenders were proposed.

These problems were previously studied under the assumption of full state information [1]–[6]. Extending these approaches to address games with incomplete state information is challenging, such as when pursuers are equipped with a limited sensing range or footprint. Such an extension has been done for more general pursuit-evasion games where the evaders do not have a specific goal location, but will move anywhere to delay capture [7], [8]. The key aspect of such studies is the searching policy to efficiently find the unseen evaders.

In the perimeter defense problem, intruders can only score by approaching the perimeter, and this suggests the defenders should search for intruders near the perimeter. We elect to cast this search as a patrolling task possibly delegated to a patroller team.

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Patrolling on a graph has been extensively studied [9], [10], where the problem reduces to the frequency of patrollers' visit to each node. On the other hand, patrolling in continuous space preserves interesting aspects of the pursuit-evasion dynamics in relation to the geometry of the game space. In [11] a patrolling path to prevent evaders from escaping a circular area was designed by posing the problem as a confinement of an expanding disk. An optimal evading strategy was explicitly considered in [12], where the critical inter-agent distance was derived to ensure that the evader cannot move between pursuers.

Once the intruders are found by the patroller subteam, the actual capture is done by the defender subteam, whose behavior is based on the results from [6]. Our interest is in the interaction between the utilities provided by the two subteams in achieving the overall defense task. This coupling motivates us to consider a team composition problem.

Team formation with heterogeneous agents have been studied with different focuses including the synergy between agents [13], the information types for the agents to interface with each other [14], and the effect of specialization [15]. There are also studies from the perspective of task allocation, where the utility of the teams are assumed to increase linearly with the number of robots [16], [17].

By looking at the concrete example of perimeter defense, we focus on two aspects that were ignored or simplified in the above works: utility models that are more general than a linear sum, and the effect of parameter uncertainty in the selection of robots.

Contributions of this paper are: (1) strategy for perimeter defense under sensing constraint; (2) relation between the team size, robot parameters, and the overall performance; and (3) policies to compose patroller and defender teams when there are parameter uncertainties.

In Sec. II we introduce a perimeter defense problem with parameter uncertainty and pose the question: How should heterogeneous agents be composed into a team? Sec. III describes a simple patrolling behavior which is leveraged in Sec. IV to build a defense strategy with parolling and defense subteams. Optimal team compositions in the presence of uncertainty are addressed in Sec. V.

II. Problem Formulation

Consider a homogeneous team of N_A intruders attempting to reach the perimeter and a defensive team attempting to prevent intrusions. We follow [6] and assume that the defenders cannot move inside the perimeter, and we restrict ourselves to convex polygonal or circular perimeters. All agents have first-order dynamics with bounded speeds. The

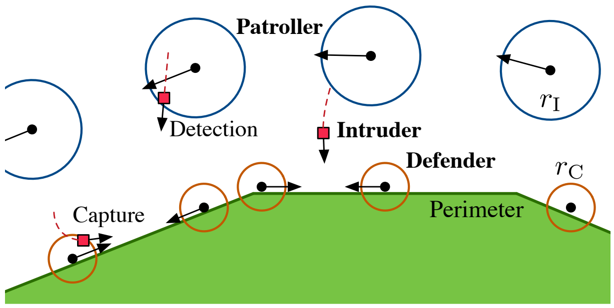

Fig. 1: Illustration of perimeter defense with patrollers and defenders.

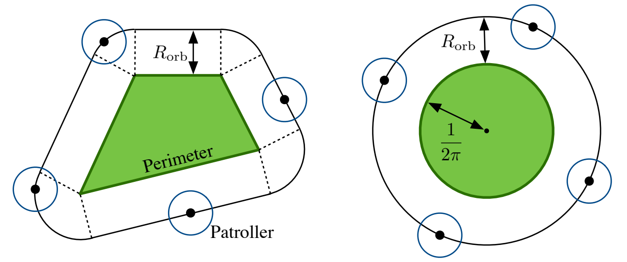

Fig. 2: Patrolling path for polygonal and circular perimeter, parameterized by R_{orb}, the distance from the perimeter.

problem can be nondimensionalized in terms of the perimeter length and the intruders' maximum speed. In the resulting nondimensional problem the perimeter length is 1, the maximum intruder speed is 1, and the defender's maximum speed is $v = v_D/v_A \geq 1$.[1]

Suppose the positions of all agents and objects near the perimeter are known *but not their identities*. This type of information might be available from a centralized sensing system such as radar which could fail to distinguish intruders from a multitude of other nonthreating agents or objects. It is inefficient or impossible to engage with every single one of those agents. The approach we consider is to use mobile robots to get close to and identify whether a given agent is a threat/intruder or not, and then to pursue it. This scenario motivates the introduction of two types of interception and the information structure defined in the following.

When the distance between a defender and an intruder is less than r_{I}, the *detection* occurs, and the defender can classify it as an intruder. Once an intruder is detected, its identity and position are known to all defenders for the rest of the game. When the distance is less than r_{C}, the *capture* occurs, and the defender can neutralize the intruder, where we assume $r_{\mathrm{C}} < r_{\mathrm{I}} < \frac{1}{2}$. The defender team aims to maximize the number of capture so that the intruder team's score (the number of intrusion) is minimized. To consider the worst-case guarantees, we assume that the intruders know the positions of all defenders, and also their strategies.

In designing the defender team, we have n types of robots that have heterogeneous capabilities in terms of the three robot parameters $\boldsymbol{\theta} = [r_{\mathrm{C}}, r_{\mathrm{I}}, v]^T$. For each robot, all three parameters may change depending on the encountered intruder robot and/or the environment. For example, the observation range r_{I} may vary due to the combination of the sensor used on the robot and the shape, size, and type of the intruder or present weather conditions. The nondimensional speed v may also vary depending on the intruder speed and environmental factors.

Suppose there are M scenarios that generate different parameter sets. We consider m to be a random variable whose sample space is $\{1, ..., M\}$, and its known probability distribution is $\mathbf{p} = [p_1, ..., p_M]^T$ where $Pr(m=k) = p_k$ and $\sum_i^M p_i = 1$. We describe the set of n robot types by $\{\boldsymbol{\Theta}_i\}_{i=1}^n$, where each $\boldsymbol{\Theta}_i \in \mathbb{R}^{3\times M}$ describes the three robot parameters in M different scenarios. The mth column $\boldsymbol{\Theta}_{i,m} \in \mathbb{R}^3$ denotes the parameters of the ith robot in the mth scenario. We can interpret the three parameters of ith robot $\boldsymbol{\theta}_i$ as a random vector whose sample space is $\{\boldsymbol{\Theta}_{i,m}\}_{m=1}^M$, and the probability distribution is $\mathbf{p}$.

[1]Note the analyses in [6] are parameterized by the reciprocal of v.

The task requirement is specified by N_A^*, the number of intruders that need to be captured. In the presence of parameter uncertainty we introduce three performance measures: the worst-case performance, the average performance, and the probability of success.

Problem (Team composition)**.** *Given (1) a set of robot types, (2) probability distribution of the robot parameters, and (3) the task requirements, design the smallest team of robots that satisfy the task requirements.*

Composing a team from a group of heterogeneous agents is difficult in general. The main challenge lies in the coupling between the selected robot team and the design of the optimal or even reasonable control policy for that specific team. We approach this problem by decomposing a *single heterogeneous-team task* into *multiple homogeneous-team subtasks*. Specifically, the perimeter defense task is decomposed into patrol and defense subtasks. The design complexity is greatly reduced because the policy design is now decoupled from the team composition.

III. Patrolling Motion

This section characterizes a simple patrolling motion that is leveraged in the next section to build defense policies.

A. Inter-Patroller Separation

For a given perimeter, we consider a family of patrolling paths parameterized by R_{orb}, the distance of the path from the perimeter (see Fig. 2). Noting that the perimeter length is 1, one cycle of the patrolling path has the length $L = 1 + 2\pi R_{\mathrm{orb}}$, for both polygonal and circular perimeters.

We restrict ourselves to the patrolling motion where uniformly spaced robots circulate in a fixed direction at their maximum speed [11]. This type of motion leads to a deterministic guarantee for interception, in contrast to the randomized motion studied in [9] which gives a probability of interception.

The following lemma presents the maximum allowable separation between the patrollers to ensure interception even when the intruder employs the optimal evasive maneuver.

Lemma 1 (Critical Inter-Patroller Separation)**.** *If any consecutive patrollers have the arc-length separation less than*

$$d_{max}(r, v) = 2rv, \tag{1}$$

then the intruder cannot move between the patrollers without being intercepted.

Proof. The result has been known for the linear part of the path [11], [18]. We provide a proof that leads to the analysis on the circular part. Consider a reference frame translating with the patrollers. The intruder wants to maximize the descent angle depicted as ϕ in Fig. 3a, and the maximum feasible angle is $\phi = \arcsin(\frac{1}{v})$ [18]. The critical inter-patroller distance is depicted in Fig. 3a, where the interception disks touch the intruder path in the patroller-fixed frame. If the patrollers are any closer than this distance, there is no intruder path that reaches the perimeter without being intercepted. From the geometry we have $\frac{1}{2}d_{\max}\sin\phi = r$, which is equivalent to $d_{\max} = 2rv$.

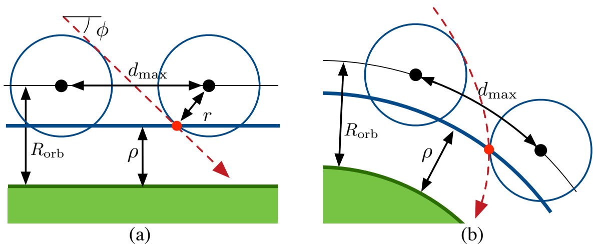

Fig. 3: The critical inter-patroller separation and the interception distance on a linear (a) and curved (b) part of the patrolling path. The red dashed lines are the optimal intruder path in the patroller-fixed frame. Note that all agents are moving towards left in the world frame.

The result on the circular portion of the patrolling path is novel. Let R denote the radius of curvature of the patrolling path: i.e., $R = R_{\text{orb}}$ for polygonal perimeters and $R = R_{\text{orb}} + \frac{1}{2\pi}$ for a circular perimeter (see Fig. 2). Instead of moving radially, the optimal intruder motion is to move at some angle away from the patroller [11], [12]. This motion results in a straight path towards the tangent point on a circle with radius $\frac{R}{v}$ in the world frame[2] [6]. In the reference frame rotating with the patrollers, this path draws a curve called an *involute*: i.e., the locus of the tip of a taut string unwrapped from the circle with radius $\frac{R}{v}$ (see [6] for details).

Consider the critical patroller positions in Fig. 4 where the interception disks touch the intruder path (involute). From the property of an involute, we know that the arc length between A and B is identical to the distance between A and C denoted as $l = R\sqrt{1-\frac{1}{v^2}} - r$. The central angle is $\angle BOA = l/(\frac{R}{v}) = \sqrt{v^2-1} - \frac{rv}{R}$. Now we obtain the azimuth of the first patroller to be $\psi_1 = \angle BOD_1 = \sqrt{v^2-1} - \frac{rv}{R} - \arccos(\frac{1}{v})$.

In a similar way, the azimuth of the second patroller can be obtained as $\psi_2 = \angle BOD_2 = \sqrt{v^2-1} + \frac{rv}{R} - \arccos(\frac{1}{v})$. The separation between the two patrollers is $\Delta\psi = \psi_2 - \psi_1 = \frac{2rv}{R}$, and the critical distance is $d_{\max} = R\Delta\psi = 2rv$. $\square$

[2] See animation at `https://youtu.be/_1IAkCAUDRY`

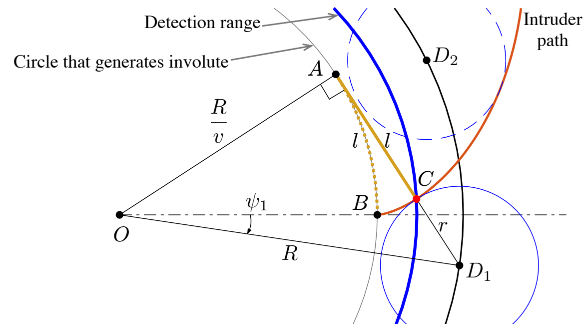

Fig. 4: Intruder path and critical patroller positions in rotating frame.

This critical inter-agent distance improves on the previous bound developed in [12] which considered only the time taken for the evader to move from radial position $R+r$ to R. This required a shorter time interval between patroller visits and led to an overly conservative finding that the required inter-agent distance be dependent on R and no greater than $\max\{2r, rv\}$, which is sufficient but not necessary.

In contrast, $d_{\max}$ in (1) is necessary and sufficient, and it is independent of the turning radius, or R_{orb}. The interception disks of neighboring patrollers need to be touching only when $v = 1$, i.e., the intruder is as fast as the patrollers.

B. Interception Distance

Next we discuss where the interception occurs.

Definition 1 (Interception Distance)**.** *The interception distance, ρ, is the minimum distance from the perimeter that the intruders can reach without being intercepted. Interception is equivalent to detection (resp. capture) when the interception disk has the radius r_I (resp. r_C).*

Suppose the number of patrollers N is given. Increasing R_{orb} would make the interception distance larger, but it also makes the inter-patroller distance larger. By solving $d_{\max}N = L = 1 + 2\pi R_{\text{orb}}$, we obtain the largest patrolling distance $R^*_{\text{orb}} = \frac{1}{2\pi}(2vrN - 1)$. Using this R^*_{orb}, we obtain the expression of ρ as a function of the robot parameters:

Lemma 2 (Guaranteed Interception Distance)**.** *A team of N patrolling robots with interception radius r and speed v provides an interception distance given by*

$$\rho(N; r, v) = r\left(\frac{Nv}{\pi} - \sqrt{1-\frac{1}{v^2}}\right) - \frac{1}{2\pi}. \tag{2}$$

To have $\rho > 0$, we need at least N_{min} patrollers:

$$N_{min}(r, v) = \left\lceil \frac{1}{v}\left(\frac{1}{2r} + \pi\sqrt{1-\frac{1}{v^2}}\right)\right\rceil. \tag{3}$$

Proof. On a linear part of the patrolling path, the geometry in Fig. 3a shows that $\rho = \rho_{\text{lin}} \triangleq R^*_{\text{orb}} - r\cos\phi = R^*_{\text{orb}} - r\sqrt{1-\frac{1}{v^2}}$, which reduces to (2). The inequality $\rho > 0$ can be solved for N to give (3).

On the circular portion, the distance between the center of rotation O and the worst-case interception point, C in Fig. 4, is obtained by $s = \sqrt{\frac{R^2}{v^2} + l^2}$, which gives

$$s(R) = \sqrt{R^2 + r^2 - 2rR\sqrt{1 - 1/v^2}}. \tag{4}$$

For polygonal perimeters, the radius of curvature is equal to R_{orb}, and the interception distance is $\rho = \rho_{\text{circ},1} \triangleq s(R^*_{\text{orb}})$. For a circular perimter, the interception distance is $\rho = \rho_{\text{circ},2} \triangleq s(R^*_{\text{orb}} + \frac{1}{2\pi}) - \frac{1}{2\pi}$. One can show that $\rho_{\text{lin}} < \rho_{\text{circ},1}$ is always true, and $\rho_{\text{lin}} < \rho_{\text{circ},2}$ is true when (3) is true. Hence, ρ_{lin} is a conservative choice on the patroller side that applies to any part of the patrolling path. □

IV. Performance Guarantees

This section derives the relationship between the robot parameters and the overall utility (guaranteed number of capture). To this end, we first discuss two defense strategies: blind defense and reactive defense. For conciseness, we restrict our discussion to a circular perimeter, but the results are extensible to polygonal perimeters with minor changes.

A. Blind Defense

A direct application of the patrolling motion in the perimeter defense is the *blind defense*, in which defenders move on the patrolling path without explicitly responding to the intruders. We assume that each defender can simultaneously capture all intruders within the capture radius. If there are sufficiently many defender robots so that the interception with radius r_{C} occurs at the distance $\rho(N; r_{\text{C}}, v) > 0$, then no intruder can reach the perimeter. The sufficient number of defenders is given by

$$N_{\text{B.D.}} \triangleq N_{\min}(r_{\text{C}}, v), \tag{5}$$

where $N_{\min}$ is defined in (3). Note that the blind defense can defend against infinitely many intruders.

B. Reactive Defense with Full State Information

This section reviews the assignment-based defense policy derived in [6], where the defenders were assumed to move on the perimeter. In contrast to the blind defense, we call this *reactive defense* because the defenders actively pursue the intruders based on the knowledge of their positions.

In the one vs. one game on a circle with radius $1/(2\pi)$, let θ and ρ denote the relative polar angle and the distance of the intruder from the perimeter. We adapt the results from [6] to define the following function:

$$\begin{aligned} V(\rho, \theta; r_{\text{C}}, v) &= -\sqrt{v^2\left(2\pi\rho + 1\right)^2 - 1} + \sqrt{v^2 - 1} \\ &+ |\theta| - 2\pi r_{\text{C}} + \cos^{-1}\left(\frac{1}{v(2\pi\rho + 1)}\right) - \cos^{-1}\left(\frac{1}{v}\right). \end{aligned} \tag{6}$$

The radius of the circle is scaled from 1 to $\frac{1}{2\pi}$, and the capture radius r_{C} is incorporated which effectively reduces the angular separation $|\theta|$. It was shown in [6] that the defender can guarantee capture with a simple feedback law (move clockwise when $\theta < 0$ and counter-clockwise when $\theta > 0$) if the initial configuration $[\rho, \theta]$ is such that $V \leq 0$. Fig. 5a shows the level set $V = 0$. Note that capture includes the situation where the intrusion is delayed indefinitely.

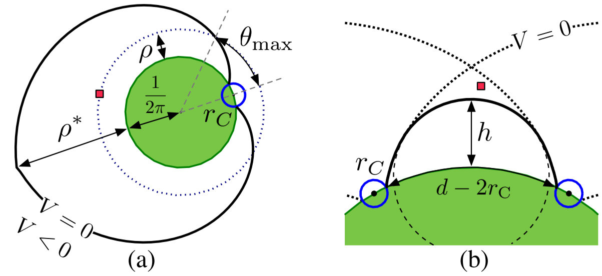

Fig. 5: Boundaries of winning regions. (a) One vs. one game. Intruder at red square can win because it is inside of the boundary (i.e., $V > 0$). (b) Two vs. one game. Individual defenders cannot capture the intruder at red square alone, but a pincer movement guarantees capture [6].

Consider the critical angle $\theta_{\max}$ which satisfies $V(\rho, \theta_{\max}) = 0$. The defender wins if the initial angular separation is less than $\theta_{\max}$. Hence, if $\theta_{\max} = \pi$, then the defender can win regardless of its location on the perimeter. Let ρ^* to be the critical distance that satisfies

$$V(\rho^*, \pi) = 0. \tag{7}$$

If the intruder starts from a distance farther than ρ^*, then the defender is guaranteed to win. Note the critical distance ρ^* is a decreasing function of both v and r_{C}.

The two vs. one game was used to show that a pincer movement employed by a pair of defenders (*cooperative defense*) enables capturing an intruder that cannot be captured by either of the defenders individually [6]. The boundary of the winning configuration is given by a combination of the level set $V = 0$ and a circle with radius $h = \frac{d - 2r_{\text{C}}}{2v}$, where d is the arc length between the two defenders (see Fig. 5b).

Given an initial configuration of the game with multiple intruders and multiple defenders, the pairwise one vs. one game analyses tell us which defender can potentially capture which intruder. Based on this information, a defender is assigned to a unique intruder only if it can win. The number of such assignments, N_{cap}, gives a lower bound on the number of intruders that will be captured (guaranteed number of capture), and the intruder's score is bounded from above by $N_A - N_{\text{cap}}$.

A combinatorial optimization to maximize N_{cap} based on maximum matching was proposed in [3]. The policy was extended in [6] to accommodate cooperative defense, i.e., a pair of defenders can be also assigned to a unique intruder.

C. Reactive Defense with Patrollers

In our game with incomplete state information, consider two subteams: patrollers and (reactive) defenders. Suppose the patrolling team provides a detection distance $\rho_I \triangleq \rho(N_{\text{I}}; r_{\text{I}}, v)$. The defender team can play the full-state-information game against a set of already detected intruders using the assignment policy reviewed in the previous section.

In [6], all the intruders start at some initial positions at the beginning of the game. Here, in the defense subtask, the intruders may appear at different time depending on when they are detected. We can consider the following initialization of the game: *the intruder team sends each intruder at a desired time from a desired azimuthal position* *with distance ρ_I from the perimeter.* (Note that this is conservative because detection may occur at farther distances.) To simplify the analysis in the sequel, we discuss how the initial configuration is generated for the defense subtask.

First, the defenders should be uniformly spaced on the perimeter when no intruder is detected yet. Noting that the intruders have the full state information, they can approach the point where the defenders are most sparse. A uniform spacing minimizes the largest gap between neighboring defenders, which justifies this formation. The situation where the intruders appear sequentially is discussed next.

Lemma 3. *Suppose the defenders employ the assignment policy, and they are uniformly spaced. If there are sufficiently many defenders so that they can capture N_A intruders that simultaneously appear together at any particular azimuthal position, then they can defend against N_A intruders that appear in any sequence (timing and/or azimuthal position).*

Proof. We provide a sketch of proof. Suppose the intruders appear sequentially. When one intruder is detected, a single defender will be assigned. Since this defender is unavailable for other intruders to be detected in the future, we can ignore it and consider a new game played between one less agent on each team. The weakest point on the perimeter is where the first defender disappeared. To fill this "hole", the rest of the defenders will reconfigure themselves into a new uniform spacing. The next intruder should approach the weakest point immediately since the gap will only shrink as the defenders converge to the new formation. Repeating this argument, we see that the intruders should appear simultaneously.

In addition, any scattering in the azimuthal position would only increase the number of defenders involved in the assignment, i.e., increase the number of capture. Therefore, the intruders should approach from a single point.

We have shown that a *simultaneous approach from a single point* is the worst-case intruder strategy that the defenders may encounter. □

This worst-case scenario (clustered intruders) simplifies the derivation of the performance guarantee presented next.

D. Utility of the Overall Team

Assuming the utility of the patroller subteam, ρ_I, the defense subteam gives the following utility:

Lemma 4 (Guaranteed Number of Capture)**.** *Suppose the intruders are detected at the distance ρ_I. If there are N_C reactive defenders with capture radius r_C and speed v, then they can defend the perimeter against N_A intruders given by*

$$N_A = \min\left\{N_C,\ \left\lfloor N_C\left(\frac{\rho_I}{\rho^*} + 2r_C\left(1 - \frac{\rho_I}{\rho^*}\right)\right)\right\rfloor\right\}, \quad (8)$$

where $\rho^* = \rho^*(r_C, v)$ is defined in (7).

Proof. First, consider the degenerate case $\rho_I > \rho^*$, where any defender can capture any intruder. Beyond this detection distance, the number of guaranteed capture is $N_A = N_C$.

When $\rho_I < \rho^*$, only some of the defenders can be assigned to the intruders. Given a detection point where N_A intruders appear simultaneously, consider the pair D_1 and D_2 shown in Fig. 6a. They have the largest separation, d_0, subject to $(\frac{d_0}{2} - r_C)/v \le \rho_I$, and among other pairs that have the same separation, their mid point is the closest to the detection point. Noting that the pair (D_1, D_2) contributes to one capture, N_A captures are guaranteed if: (i) there are at least $N_A - 1$ defenders between D_1 and D_2; and (ii) each of them contributes to one capture.

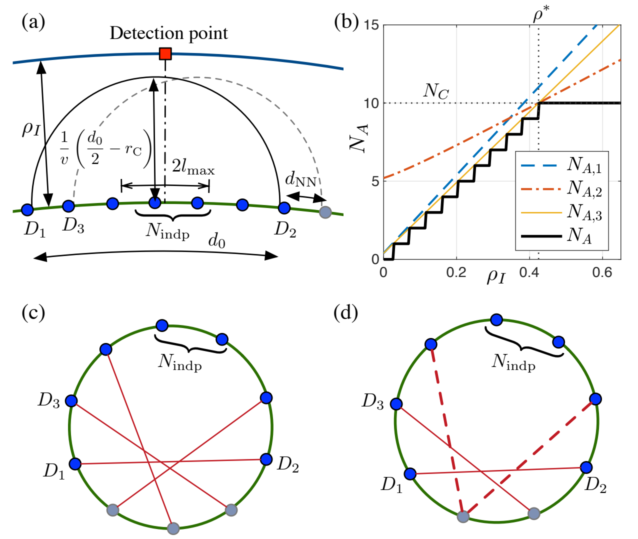

Fig. 6: Relevant quantities used in the proof of Lemma 4 (a). Illustration of N_A in the three expressions (9), (10), and (11), when $N_C = 10$ (b). The case where condition (10) is satisfied (c) and not satisfied (d). The lines indicate the pairng for cooperative defense assignments.

Let $d_{NN} = 1/N_C$ denote the nearest neighbor distance. Condition (i) is true if $\frac{d_0}{d_{NN}} \ge N_A$, which reduces to

$$N_A \le N_{A,1} \triangleq 2N_C(v\rho_I + r_C). \quad (9)$$

Now, suppose (9) is true and we have $N_A - 1$ defenders between D_1 and D_2. Consider the decomposition $N_A - 1 = N_{indp} + N_{coop}$, where N_{indp} is the number of defenders that can win individually, and N_{coop} are those who need partners from outside to perform cooperative defense (e.g., D_3 in Fig. 6a). Notice that the pairs all have the same separation (see Fig. 6c), e.g., D_3 can be paired with the first defender to the right of D_2. The N_{coop} defenders will each have a unique partner if the overall number of the defenders N_C satisfies $N_C \ge 2 + N_{indp} + 2N_{coop} = 2N_A - N_{indp}$. (See Fig. 6d for the case when this condition is not true.)

Recall $\theta_{\max}$ defined in Sec. IV-B and let $l_{\max} \triangleq \theta_{\max}/2\pi$. Then we have $N_{indp} = \lfloor \frac{2l_{\max}}{d_{NN}} \rfloor + 1 = \lfloor 2l_{\max}N_C \rfloor + 1$. Now the inequality $N_C \ge 2N_A - N_{indp}$ (i.e., condition (ii)) is equivalent to $2N_A \le N_C + 1 + \lfloor 2N_C l_{\max} \rfloor$, which is true if

$$N_A \le N_{A,2} \triangleq N_C\left(1/2 + l_{\max}\right). \quad (10)$$

The capture of N_A intruders is guaranteed if both (9) and (10) are true. For simplicity, we scale ρ in (9) to define the following quantity:

$$N_{A,3} \triangleq 2N_C(vK\rho_I + r_C), \quad (11)$$

where $K = \frac{1}{v\rho^*}\left(\frac{1}{2} - r_C\right)$. One can verify that $N_{A,3} < N_{A,1}$ always holds, and $N_{A,3} < N_{A,2}$ holds for $\rho_I < \rho^*$. Fig. 6b illustrates the three different conditions. Rearranging the expression $\lfloor N_{A,3} \rfloor$ gives the second term in (8). □

The implicit contribution of the defender speed v appears in ρ^*, which is a decreasing function of v (Sec. IV-B). The second term in the $\min$ operator increases with ρ_I, implying that more defender can be assigned to the intruders as they appear farther from the perimeter. The second term matches N_{C} when $\rho_I = \rho^*$. Let N^* be defined as

$$N^* \triangleq \min N_{\mathrm{I}} \ \text{ s.t. } \ \rho(N_{\mathrm{I}}; r_{\mathrm{I}}, v) \geq \rho^*, \tag{12}$$

which is computed using (2). When $N_{\mathrm{I}} \geq N^*$, or equivalently $\rho_I \geq \rho^*$, any defender can capture any intruder, so the number of guaranteed capture equals the number of defenders. In other words, there is no incentive to increase the detection range ρ_I beyond ρ^*.

The patroller and defender subteams work together to give the overall utility. The form of cooperation here is *service provider and client*, i.e., the patroller team provides the "detection service" quantified by ρ_I, which is necessary for the defender team to complete their subtask. The overall utility is summarized in the following:

Lemma 5 (Overall Utility)**.** *Given N_I patroller robots with $\boldsymbol{\theta}_I = [r_I, r'_C, v_I]^T$ and N_C defender robots with $\boldsymbol{\theta}_C = [r'_I, r_C, v_C]^T$, they can defend the perimeter against $N_A = \lfloor U \rfloor$ intruders with U defined by*

$$U(N_I, N_C; \boldsymbol{\theta}_I, \boldsymbol{\theta}_C) \triangleq \begin{cases} \infty & \text{if } N_C \geq N_{B.D.} \\ N_C & \text{elseif } N_I \geq N^* \\ N_C(\alpha N_I + \beta) & \text{otherwise,} \end{cases} \tag{13}$$

where

$$\begin{aligned} \alpha &= \frac{(1-2r_C) v_I r_I}{\rho^* \pi}, \quad \textit{and} \\ \beta &= 2r_C - \frac{1-2r_C}{\rho^*}\left(r_I \sqrt{1 - \frac{1}{v_I^2}} + \frac{1}{2\pi} \right). \end{aligned}$$

Proof. The first case is blind defense (5), which can defend against infinitely many intruders. The second case is when the patroller team provides $\rho_I > \rho^*$. The third case follows from (2) and (8). □

Observe how the utility U changes nonlinearly with respect to the overall teams size $N = N_{\mathrm{I}} + N_{\mathrm{C}}$.

V. Team composition

Given a set of available robots and the uncertainty in the parameters, this section finds the smallest team that satisfies the task requirements.

A. Deterministic Case

We first consider the case where there is no uncertainty in the parameters ($M = 1$). The parameter matrix $\boldsymbol{\Theta}_i \in \mathbb{R}^{3\times M}$ reduces to a vector $\boldsymbol{\theta}_i \in \mathbb{R}^3$.

Algorithm 1 Robot type selection (Deterministic)

```
1: Select patroller robot type that maximizes r_I v
2: Initialize N_min = ∞
3: for every n robot type do                    ▷ Find defender robot
4:     Find (N_I, N_C) that minimizes N_I + N_C subject to U ≥ N*_A
5:     N_tot ← min{N_I + N_C, N_B.D.}
6:     if N_tot < N_min then
7:         save robot type and N_min ← N_tot
8: return robot type
```

1) Team size: Suppose a specific set of robot types $\boldsymbol{\theta}_{\mathrm{I}}$ and $\boldsymbol{\theta}_{\mathrm{C}}$ is given. We first discuss how to find the pair $(N_{\mathrm{I}}, N_{\mathrm{C}})$ that minimizes $N_{\mathrm{tot}} = N_{\mathrm{I}} + N_{\mathrm{C}}$ subject to $U \geq N_A^*$, where N_A^* is the task requirement. We consider three candidate teams in the following and select the smallest team.

The first candidate is the blind defender team: $N_{\mathrm{I}} = 0$ and $N_{\mathrm{C}} = N_{\mathrm{B.D.}}(\boldsymbol{\theta}_{\mathrm{C}})$. The second candidate is when the patroller team provides a sufficiently large detection distance: i.e., $N_{\mathrm{I}} = N^*(\boldsymbol{\theta}_{\mathrm{I}})$ and $N_{\mathrm{C}} = N_A^*$. The third candidate is the general case where we find $(N_{\mathrm{I}}, N_{\mathrm{C}})$ such that $N_{\mathrm{C}}(\alpha N_{\mathrm{I}} + \beta) \geq N_A^*$. An exhaustive search through all combination of N_{I} and N_{C} only grows quadratically with the size of the final solution N_{tot}. In addition, the search can be stopped when N_{tot} reaches $N_{\mathrm{B.D.}}$.

2) Robot type selection: Now we consider how to select the robot type for each subtask. Recall the utility of the patroller team given in (2), which can be approximated by

$$\rho_I(N_{\mathrm{I}}; r_{\mathrm{I}}, v) \approx \frac{N_{\mathrm{I}} r_{\mathrm{I}} v}{\pi} - \frac{1}{2\pi}$$

when N_I is sufficiently large. Finding the optimal patroller robot type is straightforward since we can simply use $r_{\mathrm{I}} v$ as a metric, and choose the robot type that has the maximum value. Importantly, this selection is independent from the defense subtask.

The selection of the optimal defender robot type is more ambiguous. Recalling (8) the relative importance of robot speed v and the capture radius r_{C} is not obvious, and it changes as a function of ρ_I. The smaller ρ_I is (i.e., it takes shorter time for the intruder to reach the perimeter), the more important r_{C} is.

Exhaustive search by pairing every robot type with the selected patroller type is feasible because the complexity increases only linearly with the number of robot types, n. The search is summarized in Algorithm 1.

B. Task Requirements under Uncertainty

We now consider the selection of the robot type when the robot parameters are random variables. Given a team of patrollers and defenders, the utilities under M different scenarios can be computed using (13), and we summarize it in a vector $\mathbf{U} \in \mathbb{R}^M$. Depending on the application, whether or not this team satisfies the task requirement can be considered in different ways:

1) Worst case guarantee: Consider a binary vector $\mathbf{s}$ whose ith element is 1 if $[\mathbf{U}]_i \geq N_A^*$ and 0 otherwise, where N_A^* is the minimum required number of capture. Let $\mathbf{1}$ denote a vector of ones with appropriate dimension. The given team succeeds in all scenario if

$$\mathbf{s} = \mathbf{1}. \tag{14}$$

2) Probability of success: The probability of success is given by the inner product of $\mathbf{s}$ and $\mathbf{p}$. We define success by

$$\mathbf{s} \cdot \mathbf{p} \geq P^*, \tag{15}$$

where P^* is the threshold specified in the task requirement.

3) Expected performance: The expected number of guaranteed capture is given by the inner product of $\mathbf{U}$ and $\mathbf{p}$. Success is defined by

$$\mathbf{U} \cdot \mathbf{p} \geq N_A^*. \tag{16}$$

The quantity $\mathbf{U} \cdot \mathbf{p}$ describes how much, on average, the intruder team's score is reduced. We later discuss when to use blind defense, which addresses the case when some elements of $\mathbf{U}$ are ∞.

C. Optimal team

We find the smallest team (i.e., robot types and their number) that satisfies the task requirement. The types of the patrolling robot $\mathbf{\Theta}_I$ and the defense robot $\mathbf{\Theta}_C$ needs to be selected from the set of available robots $\{\mathbf{\Theta}_i\}_{i=1}^n$, to solve the following optimization problem:

$$\begin{aligned} \text{minimize} \quad & N_{\text{tot}} = N_{\text{I}} + N_{\text{C}} \\ \text{subject to} \quad & \mathbf{U}(N_{\text{I}}, N_{\text{C}}; \mathbf{\Theta}_I, \mathbf{\Theta}_C) \\ & \text{satisfying (14), (15), or (16).} \end{aligned} \tag{17}$$

The constraint is selected depending on the application.

Unlike the deterministic case, even the optimal patroller team cannot be selected easily, due to the interaction with the defender team. Recalling (8), the same overall utility can be achieved either by using a strong patroller team (i.e., large ρ_I) or by using a strong defender team (i.e., large N_{C}, r_{C}, or v_{C}). Suppose the task requirement is the worst-case performance. A good combination of patroller and defender robots would have parameter variations that augment each other: i.e., in a case when one team has a disadvantageous parameter set, the other team has a favorable parameter set.

For example, consider the parameter set given in Fig. 7. All the robots have the same r_{C}. The optimal (smallest) team to defend against $N_A^* = 5$ with constraint (14) turns out to be the combination of 53 yellow robots for patrolling and 16 blue robots for the defense subtask. The election of the blue robot may be surprising noting that it is inferior than the red robot in both worst-case and average speeds. Nevertheless, it is selected because in scenario 2 it can augment the deteriorated speed of yellow robot better than the red robot.

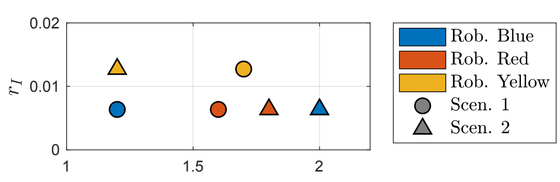

Fig. 7: Example parameter set with $n = 3$ and $M = 2$. The color represents the robot type and the shape represents the scenario. The capture radii are $r_{\text{C}} = 0.003$ for all robots.

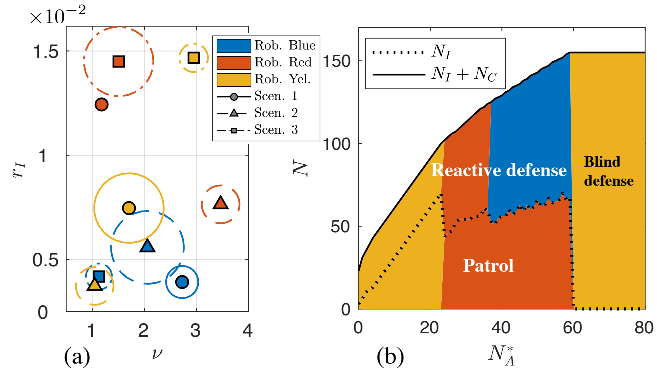

Fig. 8: (a) The parameters of three robots in three scenarios. The radii of the circles are proportional to r_{C}. The probability distribution is selected to be $\mathbf{p} = \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right]^T$. (b) The size and the type of robots used in the best team, where the criterion is the expected performance.

When the number of robot types n is small, exhaustive search summarized in Algorithm 2 is feasible since the complexity only scales quadratically with n. From line 4,

Algorithm 2 Robot type selection (Parameter uncertainty)

1: Initialize $N_{\text{min}} = \infty$
2: **for** all combination (i, j), where $i, j \in \{1, ..n\}$ **do**
3: Solve (17) with $\mathbf{\Theta}_I \leftarrow \mathbf{\Theta}_i$ and $\mathbf{\Theta}_C \leftarrow \mathbf{\Theta}_j$
4: $N_{\text{B.D.}} \leftarrow \max_{m \in \{1,..M\}} N_{\text{B.D.}}(\mathbf{\Theta}_{j,m})$
5: $N_{\text{tot}} \leftarrow \min\{N_{\text{I}} + N_{\text{C}}, N_{\text{B.D.}}\}$
6: **if** $N_{\text{tot}} < N_{\text{min}}$ **then**
7: types $\leftarrow (i, j)$ and $N_{\text{min}} \leftarrow N_{\text{tot}}$
8: **return** types

notice that we use the worst-case $N_{\text{B.D.}}$ as the representative for the blind defense. This is because employing a blind defense with insufficient number of defenders does not guarantee any capture when the intruders are aware of the employed strategy and use the optimal evasive maneuver (see the proof of Lemma 1). To avoid such catastrophic failure, the blind defense should be used only when it is valid in all M scenarios. In addition, this restriction ensures that elements of $\mathbf{U}$ are all finite or all infinite.

Fig. 8a shows a randomly generated example of the parameters when we have three robots ($n = 3$) and three scenarios ($M = 3$). Fig. 8b shows the smallest team satisfying (16) with the task requirement N_A^* varied. The colors depict the robot type; lower part is the patroller and the upper part is the defender robot. For $25 < N_A^* < 35$ we see homogeneous red robots being allocated to both patrol and defense, for $35 < N_A^* < 60$ we see red robots patrolling and blue robots defending, and beyond 60 we see homogeneous yellow robots all doing a blind defense.

D. Approximation method

When we have a large number of robot types, we propose to reduce the complexity by choosing the patroller type

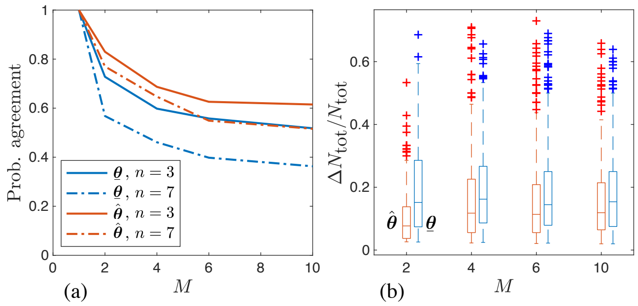

Fig. 9: Performance of the approximated robot selection. (a) Probability of agreement with exhaustive search. (b) Boxplot illustrating the suboptimality in the total number of agents for the instances that disagreed with exhaustive search, for $n = 7$. The red (resp. blue) is the data from averaged (resp. worst-case) parameter.

independently from the defender type. This is done by defining an approximation of the metric $r_{\text{I}}v$ used in Sec. V-A. The *average parameter* $\hat{\boldsymbol{\theta}}_i$ is defined by

$$\hat{\boldsymbol{\theta}}_i = \boldsymbol{\Theta}_i \mathbf{p}, \tag{18}$$

and the *worst-case parameter* $\underline{\boldsymbol{\theta}}_i$ is defined by

$$[\underline{\boldsymbol{\theta}}_i]_k = \min_m [\boldsymbol{\Theta}_{i,m}]_k. \tag{19}$$

Note that $\underline{\boldsymbol{\theta}}_i$ may not represent any of the M scenarios because the $\min$ operation is performed element wise.

These approximations reduce the selection problem to $M = 1$ (deterministic case), for which the complexity only grows linearly with n. After fixing the patroller type, we search through all n candidates to find the best defender type. In doing so, the optimization (17) is solved with the original parameter set, so the final solution $(N_{\text{I}}, N_{\text{C}})$ has a guarantee to satisfy the task requirement. The only approximation is in the robot type selection which may lead to larger team sizes.

Fig. 9 shows the performance of the approximated methods, where the worst-case guarantee (14) with $N_A^* = 10$ was used as the task criterion. We randomly generated 1000 parameter sets for each n and M value with the following bounds: $r_{\text{I}} \in [0.2, 2]/L$, $r_{\text{C}} \in [5, 15]/L$ and $v \in [1, 3]$, where $L = 300$. Fig. 9a shows how often the robot selection agreed with the optimal one from exhaustive search. The agreement tend to degrade as the number of scenarios, M, and the robot types, n, increase. For the task requirement (14), using the averaged parameter $\hat{\boldsymbol{\theta}}_i$ leads to the correct selection of the patrolling robot more often than when $\underline{\boldsymbol{\theta}}_i$ is used.

Fig. 9b shows the statistics of the differences in the total number of robots N_{tot} when the robot selection disagreed with the exhaustive search. The results from $n = 7$ is shown. The figure shows that suboptimal choices of robot type may lead to a significant increase in the required number of robots.

VI. Conclusion

We study perimeter defense problem where the intruders first need to be detected before they can be pursued. A simple patrolling motion is characterized to find the critical inter-patroller distance that ensures detection. The utility of the patroller subteam is derived as the distance from the perimeter that the intruders are guaranteed to be detected. The patroller subteam is combined with the defender subteam to complete the perimeter defense task. The coupling between the performances of the subteams motivated the team composition problem. We showed how the parameter uncertainty makes the robot selection non-trivial, and proposed an approximation method to simplify the search. Through the examples, we showed how a heterogeneous team can be efficiently constructed as a combination of homogeneous teams.

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